Name:_______________________________ Date:____________________________

HW Pre-Calculus 11 Section 5.2 Multiplying Dividing and Rationalizing with Radicals

1. Multiply each of the following radicals:

a) $\sqrt{24}\times\sqrt{6}$ 12	b) $3\sqrt{12}\times5\sqrt{8}$ $60\sqrt{6}$	c) $5\sqrt[3]{50}\times3\sqrt[3]{60}$ $150\times\sqrt[3]{3}$
d) $-4\sqrt[3]{-100}\times2\sqrt[3]{54}$ $48\sqrt[3]{25}$	e) $\sqrt[3]{a^2bc^3}\times\sqrt[3]{a^5b^4c^2}$ $a^2bc\sqrt[3]{ab^2c^2}$	f) $\sqrt[4]{32x^3y}\times\sqrt[4]{64x^2y^7}$ $4xy^2\left(\sqrt[4]{8x}\right)$
g) $2\sqrt{3}\left(4\sqrt{21}+5\sqrt{15}\right)$ $24\sqrt{7}+30\sqrt{5}$	h) $4\sqrt{5}\left(6\sqrt{40}+3\sqrt{50}-2\sqrt{90}\right)$ $120\sqrt{2}+60\sqrt{10}$	i) $5\sqrt{6}\left(4\sqrt{24}-3\sqrt{48}-5\sqrt{54}\right)$ $-210-180\sqrt{2}$

j) $\left(3\sqrt{2}+4\sqrt{3}\right)\left(5\sqrt{3}-\sqrt{8}\right)$ $15(\sqrt{6})-3(\sqrt{24})+20(3)-4\sqrt{24}$ $15\sqrt{6}-3(2)\sqrt{6}+60-4(2)\sqrt{6}$ $15\sqrt{6}-6\sqrt{6}-8\sqrt{6}+60$ $60-\sqrt{6}$ //	k) $\left(\sqrt{6}-\sqrt{8}\right)\left(\sqrt{2}+\sqrt{5}+4\right)$ $\sqrt{12}+\sqrt{30}+4\sqrt{6}-\sqrt{16}-\sqrt{40}-4\sqrt{8}$ $2\sqrt{3}+\sqrt{30}+4\sqrt{6}-4-2\sqrt{10}-8\sqrt{2}$
L) $\left(3\sqrt[3]{8x^2}+\sqrt[3]{4x^2}\right)\left(\sqrt[3]{2x^2}-6\sqrt[3]{8x^2}\right)$ $3\sqrt[3]{16x^4}-18\sqrt[3]{64x^4}+\sqrt[3]{8x^4}-6\sqrt[3]{32x^4}$ $3\sqrt[3]{2^4x^4}-18\sqrt[3]{2^6x^4}+\sqrt[3]{2^3x^4}-6\sqrt[3]{2^5x^4}$ $6x\sqrt[3]{2x}-72x\sqrt[3]{x}+2x\sqrt[3]{x}-12x\sqrt[3]{4x}$	m) $\left(8a-6\sqrt[3]{3r}\right)\left(2\sqrt[3]{18r^2}+4\sqrt[3]{45r}\right)$

2. Divide and Rationalize each of the following radicals:

a) $\dfrac{\sqrt{24}}{\sqrt{3}}=\dfrac{\sqrt{8}\times\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$ $=2\sqrt{2}$	b) $\dfrac{3\sqrt{20}}{2\sqrt{10}}=\dfrac{3\sqrt{2}\times\cancel{\sqrt{10}}}{2\cancel{\sqrt{10}}}=\dfrac{3\sqrt{2}}{2}$	c) $\dfrac{3\sqrt{18}}{5\sqrt{24}}=\dfrac{3\sqrt{3}\times\cancel{\sqrt{6}}}{5\sqrt{4}\times\cancel{\sqrt{6}}}$ $=\dfrac{3\sqrt{3}}{5\times2}$ $=\dfrac{3\sqrt{3}}{10}$ //

$d)\ \frac{1}{\sqrt{5}}-\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt5}\cdot\frac{\sqrt5}{\sqrt5}-\frac{1}{\sqrt3}\cdot\frac{\sqrt3}{\sqrt3}$ $\frac{\sqrt5}{5}-\frac{\sqrt3}{3}=\frac{3\sqrt5-5\sqrt3}{15}$	$e)\ \frac{1}{\sqrt{3}}+\frac{2}{\sqrt{6}}$ $\frac{\sqrt3}{3}+\frac{2\sqrt6}{6}$ $=\frac{\sqrt3}{3}+\frac{\sqrt6}{3}=\frac{\sqrt3+\sqrt6}{3}$ //	$f)\ \frac{5}{\sqrt{5}}-\frac{8}{\sqrt{2}}$ $\frac{5\sqrt5}{5}-\frac{8\sqrt2}{2}$ $\sqrt5-4\sqrt2$ //
$g)\ \frac{3\sqrt{48}}{2\sqrt{75}}-\frac{2\sqrt{24}}{\sqrt{96}}$ $\frac{3\sqrt{16}\times\cancel{\sqrt3}}{2\sqrt{25}\times\cancel{\sqrt3}}-\frac{2\cancel{\sqrt{24}}}{\cancel{\sqrt{24}}\sqrt4}$ $=\frac{12}{10}-\frac{2}{\sqrt4}=\frac{6}{5}-1$ $=\frac{1}{5}$ //	$h)\ \frac{3\sqrt{5}}{\sqrt{20}}+\frac{4\sqrt{3}}{\sqrt{27}}$ $\frac{3\cancel{\sqrt5}}{\sqrt4\cancel{\sqrt5}}+\frac{4\cancel{\sqrt3}}{\sqrt9\cancel{\sqrt3}}$ $\frac{3}{2}+\frac{4}{3}$ $=\frac{9+8}{6}=\frac{17}{6}$	$i)\ \frac{2\sqrt{3}}{\sqrt{9}}-\frac{3\sqrt{5}}{\sqrt{125}}$ $\frac{2\sqrt3}{3}-\frac{3\cancel{\sqrt5}}{\cancel{\sqrt5}\sqrt{25}}$ $=\frac{2\sqrt3}{3}-\frac{3}{5}$ $=\frac{10\sqrt3-9}{15}$ //
j) $\frac{1}{\sqrt{2}-\sqrt{3}}\cdot\frac{(\sqrt2+\sqrt3)}{(\sqrt2+\sqrt3)}$ $=\frac{\sqrt2+\sqrt3}{2\cancel{-\sqrt6+\sqrt6}-3}$ $=\frac{\sqrt2+\sqrt3}{-1}=-\sqrt2-\sqrt3$	k) $\frac{2}{2\sqrt{3}+5}$	L) $\frac{\sqrt{2}}{2\sqrt{3}+\sqrt{5}}\cdot\frac{(2\sqrt3-\sqrt5)}{(2\sqrt3-\sqrt5)}$ $=\frac{2\sqrt6-\sqrt{10}}{12-2\sqrt{15}+2\sqrt{15}-5}$ $=\frac{2\sqrt6-\sqrt{10}}{7}$ //
m) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}$	n) $\frac{5\sqrt{3}}{2\sqrt{2}-3\sqrt{3}}\cdot\frac{(2\sqrt2+3\sqrt3)}{(2\sqrt2+3\sqrt3)}$ $=\frac{10\sqrt6+45}{8-6\sqrt6+6\sqrt6-27}$ $=\frac{10\sqrt6+45}{-19}$	p) $\frac{x^4+x^2}{\sqrt{x^3}}\cdot\frac{\sqrt x}{\sqrt x}$ $=\frac{(x^4+x^2)\sqrt x}{\sqrt{x^4}}$ $=\frac{\cancel{x^2}(x^2+1)\sqrt x}{\cancel{x^2}}$ $=x^2\sqrt x+\sqrt x$ //
Q) $\frac{5}{\sqrt[3]{x^2}}$ Rationalize it by multiplying what the denominator needs to be a perfect cube: $x^2\times x=x^3$	R) $\frac{\sqrt[3]{3}+4\sqrt[3]{3}}{\sqrt[3]{3^2}}$	s) $\frac{\sqrt[4]{6}-3\sqrt[4]{6}}{\sqrt[4]{216}}$ When rationalizing a fourth root, we need to multiply the denominator by a value that makes it a power of 4 $216\times6=6^4$

$\frac{5}{\sqrt[3]{x^2}}\frac{\sqrt[3]{x}}{\sqrt[3]{x}}$ $=\frac{5\sqrt[3]{x}}{\sqrt[3]{x^3}}$ $=\frac{5\sqrt[3]{x}}{x}$	$\frac{\sqrt[3]{3}+4\sqrt[3]{3}}{\sqrt[3]{3^2}}=\frac{5\sqrt[3]{3}}{\sqrt[3]{3^2}}$ $=\frac{5\sqrt[3]{3}}{\sqrt[3]{3^2}}\times\frac{\sqrt[3]{3}}{\sqrt[3]{3}}$ $=\frac{5\sqrt[3]{9}}{\sqrt[3]{3^3}}=\frac{5\sqrt[3]{9}}{3}$	$\frac{\sqrt[4]{6}-3\sqrt[4]{6}}{\sqrt[4]{216}}=\frac{-2\sqrt[4]{6}}{\sqrt[4]{216}}$ $\frac{-2\sqrt[4]{6}}{\sqrt[4]{216}}\times\frac{\sqrt[4]{6}}{\sqrt[4]{6}}$ $\frac{-2\sqrt[4]{36}}{\sqrt[4]{6^4}}=\frac{-2\sqrt[4]{36}}{6}$ $=\frac{-\sqrt[4]{36}}{3}$

3. Is the following statement true or false? Explain: $\sqrt{-3}\times\sqrt{-27}=9$

   Technically, the square of a negative is not "undefined" but is not a real number anymore, and it considered an "imaginary number" $\sqrt{-1}=i$, $\sqrt{-3}=\sqrt{3}i$, and $\sqrt{-27}=\sqrt{27}i$, where "i" is an imaginary value such that: $i\times i=-1$
   So using this information the expression above becomes:
   $=\sqrt{-3}\times\sqrt{-27}$
   $=\sqrt{3}i\times\sqrt{27}i$
   $=\sqrt{81}i^2$
   $=9\times(-1)$
   $=-9$

4. The following student rationalized the expression with the steps shown. Indicate any errors that you see:

$$\frac{5-\sqrt{a}}{\sqrt{a}-4}=\frac{5-\sqrt{a}}{\sqrt{a}-4}\frac{\times\left(\sqrt{a}+4\right)}{\times\left(\sqrt{a}+4\right)}$$

$$=\frac{5\sqrt{a}-a+20}{a-4}\rightarrow \text{This step is wrong because it isn't FOILed correctly!!}$$

$$=\frac{5\sqrt{a}+20}{-4}\rightarrow \text{This step is wrong because you cant cancel out the "a"}$$

Correct method:

$$\frac{5-\sqrt{a}}{\sqrt{a}-4}=\frac{5-\sqrt{a}}{\sqrt{a}-4}\frac{\times\left(\sqrt{a}+4\right)}{\times\left(\sqrt{a}+4\right)}$$

$$=\frac{5\sqrt{a}-a+20-4\sqrt{a}}{a-16}$$

$$=\frac{\sqrt{a}-a+20}{a-16}$$

5. Find the unknown value "K" in each of the following expressions:

a) $K \times 3\sqrt{24} = 2\sqrt{3} \times 6\sqrt{10}$

Multiply the terms on the right and then simplify

$K \times 3\sqrt{24} = 12\sqrt{30}$

$K \times 3 \times 2\sqrt{6} = 12\sqrt{30}$

$K \times 6\sqrt{6} = 12\sqrt{30}$

$K = 2\sqrt{5}$

b) $8\sqrt{3} = \dfrac{4\sqrt{48}}{\sqrt{K}}$ Cross multiply to isolate the value of "k"

$\sqrt{K} = \dfrac{4\sqrt{48}}{8\sqrt{3}}$

$\sqrt{K} = \dfrac{\sqrt{16}}{2}$

$\sqrt{K} = 2$

$K = 4$

6. Find the volume of a box given the dimensions: Height: $3\sqrt{2}+4$, Width: $4\sqrt{5}-2\sqrt{3}$, Length: $4\sqrt{5}+2\sqrt{3}$
Just multiply the length width and height to get the volume:

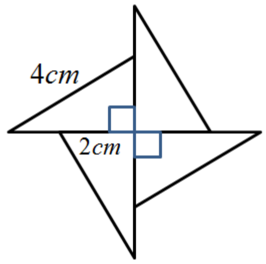
$V = \left(3\sqrt{2}+4\right)\left(4\sqrt{5}-2\sqrt{3}\right)\left(4\sqrt{5}+2\sqrt{3}\right)$

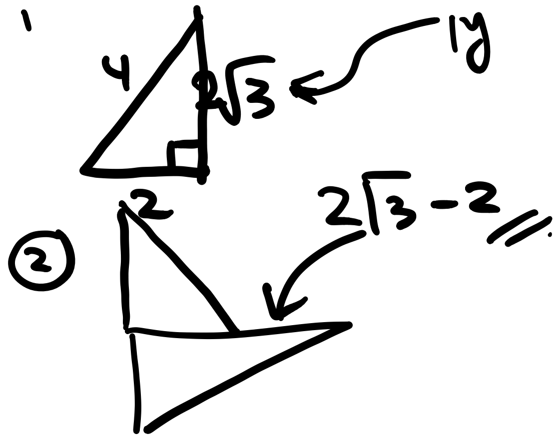
$V = \left(3\sqrt{2}+4\right)\left(80-12\right)$

$V = \left(3\sqrt{2}+4\right)68$

$V = 204\sqrt{2}+272$

7. Each right triangle in the figure shown has a hypotenuse 4cm and the shortest side 2 cm. Find the perimeter of the figure: O ? tha rean th





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$$P = (4 + 2\sqrt{3} - 2)4$$
$$= (2 + 2\sqrt{3})4$$
$$= 8 + 8\sqrt{3}$$ //

8. Challenge: Find the sum of the expression without a calculator:

$$\frac{1}{3+2\sqrt{2}} + \frac{1}{2\sqrt{2}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{5}+2} + \frac{1}{2+\sqrt{3}}$$

Rationalize each one separately and look for the pattern:

$$\frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} = \frac{3-2\sqrt{2}}{1} = 3-2\sqrt{2}$$

$$\frac{1}{2\sqrt{2}+\sqrt{7}} \times \frac{2\sqrt{2}-\sqrt{7}}{2\sqrt{2}-\sqrt{7}} = \frac{2\sqrt{2}+\sqrt{7}}{8-7} = \frac{2\sqrt{2}+\sqrt{7}}{1} = 2\sqrt{2}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}-\sqrt{6}}{7-6} = \frac{\sqrt{7}-\sqrt{6}}{1} = \sqrt{7}-\sqrt{6}$$

Can you see a pattern yet??

